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A BAYESIAN TECHNIQUE FOR SOLVING ELECTROMAGNETIC NDE INVERSE PROBLEMS

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INTRODUCTION

Determination of flaws/cracks from measurements is a common inverse problem in electromagnetic nondestructive evaluation (NDE). This inverse problem is ill-posed due to non-uniqueness of the solution, particularly in the presence of measurement noise. This paper proposes a novel state-space approach to combat the ill-posedness in the solution to the inverse problem. In particular, the problem is modeled using two equations – a state transition equation and a measurement model – that are used to model the evolution of the system with time:

$$x_k = f_{k-1}(x_{k-1}) + v_{k-1}$$
$$zk = h_k(x_k) + w_k \qquad (1)$$

where x_k represents the depth of the flaw at location k, z_k is the corresponding measurement, f_{k-1} and h_k are known non-linear functions, and v_{k-1} and w_k represent the process noise and measurement noise respectively.

Under this model, the inverse problem reduces to determining the posterior probability $p(\mathbf{x} \mid \mathbf{z})$. A recursive approach is used in this paper to evaluate the posterior probability. In particular, the state-space model resembles the classical discrete-time tracking problem, and enables the application of Bayesian non-linear filters (also known as particle filters) for the solution. Particle filters are sequential Monte Carlo (MC) methods based on point mass (or "particle") representation of probability densities. The required posterior probability may be computed from these density functions.

The proposed approach requires specification of the state vector of the system, as well as defining appropriate state and measurement models. This paper defines the state of the system as a vector of flaw depths at adjacent spatial locations:

$$\mathbf{x}(p) = \left\{ d_{p-L}, d_{p-L+1}, d_{p-L+2}, \ldots, d_p, d_{p+1}, d_{p+2}, \ldots d_{p+L} \right\} \qquad (2)$$

where p is an index over space, and d_k corresponds to the depth at location k. The state vector $\mathbf{x}(p)$ is of length $2L+1$, i.e., it contains depth information in a window of size $2L+1$ around the point p. In addition to the state vector, the proposed approach uses a numerical model derived from the physics of the NDE inspection process as the measurement model and a probabilistic state transition model. In this paper, the NDE measurements are assumed to be eddy current measurements. [1-2]

NEUMERICAL MODEL AND FIELD FORMULATION

A numerical model is used to represent the eddy current measurement process (i.e. to solve the forward problem) [2-3]. An alternating current carrying coil (source coil) along with a measurement coil are moved across a stainless-steel sample ($\sigma = 0.98 \times 10^6$ S/m). The coils are displaced along the length of the specimen and measurements are taken at every position of the coils. The electric field measured by the measurement coils may be formulated as follows. The incident field E^{inc} in the region of the sample due to a source current density is given by

$$E^{inc} = j.\omega\mu \int_s g_{12}(r - r').J(r')dx'dy' \qquad (3)$$

where g_{12} is the Green's function given by

$$g_o(r - r') = j/4H_o(k(|r - r'|) \qquad (4)$$

The incident field interacts with any flaw in the specimen, and an equivalent current density $J_d(r')$ may be computed using

$$E_o(r) = E_d(r) - k_2^2 \int_s g_{22}(r - r').J_d(r')dx'dy' \qquad (5)$$

where E_0 is the field existing in the absence of defect (the incident field) and E_d is the field in the presence of the defect (the total field).The Green's function g_{22} indicates interactions within the sample. This equation may be solved using the method of moments. Finally, the field measured at the measurement coil location due to current $J_d(r')$ is given by

$$E^{meas} = j.\omega\mu \int g_{12}(r - r').J_d(r')dx'dy' \qquad (6)$$

PARTICLE FILTERS FOR INVERSE PROBLEM SOLUTION

The key idea of particle filter technique is to represent the required posterior density function by a set of random samples with associated weights and to compute estimates based on samples and weights [3]. The weights are chosen using the principle of importance sampling [4]. As the number of samples becomes very large, this MC characterization becomes an equivalent representation to the usual functional description of the posterior PDF, and the filter approaches the optimal Bayesian estimate. A variant of particle filter called the Sampling Importance Re-sampling (SIR) filter is employed in this paper.

RESULTS

Results of applying the proposed technique will be presented at the symposium.

REFERENCES

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